Hello

Advanced Analog Integrated Circuits

Electronic Noise

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Types of Noise

- Interference, “Man-made” Noise
  - Substrate coupling
  - Supply noise
  - Signal coupling
  - Solutions:
    - Fully differential circuits
    - Layout techniques
    - Shielding
- Electronic noise
  - Fundamental physics based
    - Thermal and shot noise ← Focus of this discussion
  - Technology related
    - Flicker noise (see later), drift

Noisy Signals

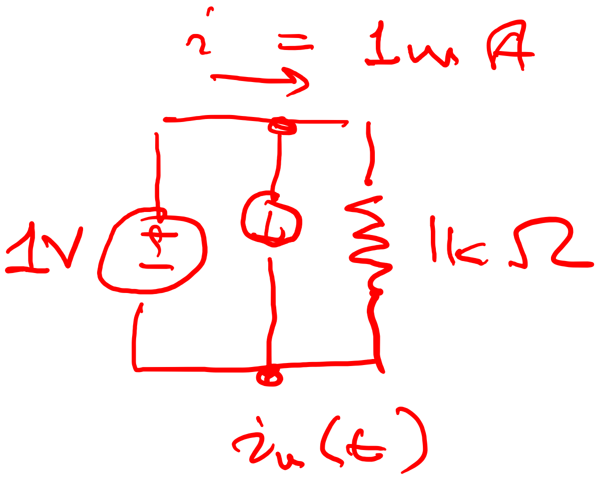
Information = $\boxed{DR} \cdot BW$

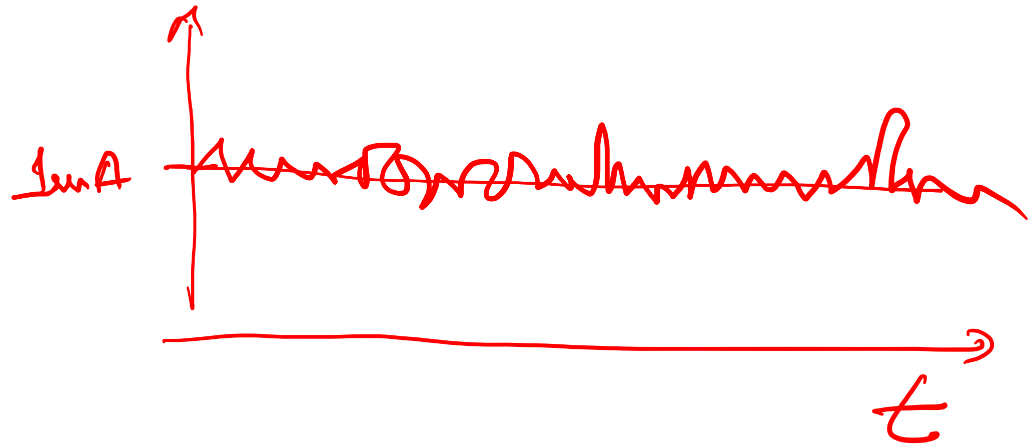
↑

thermal noise

Thermal Noise Manifestations

Example: Resistor

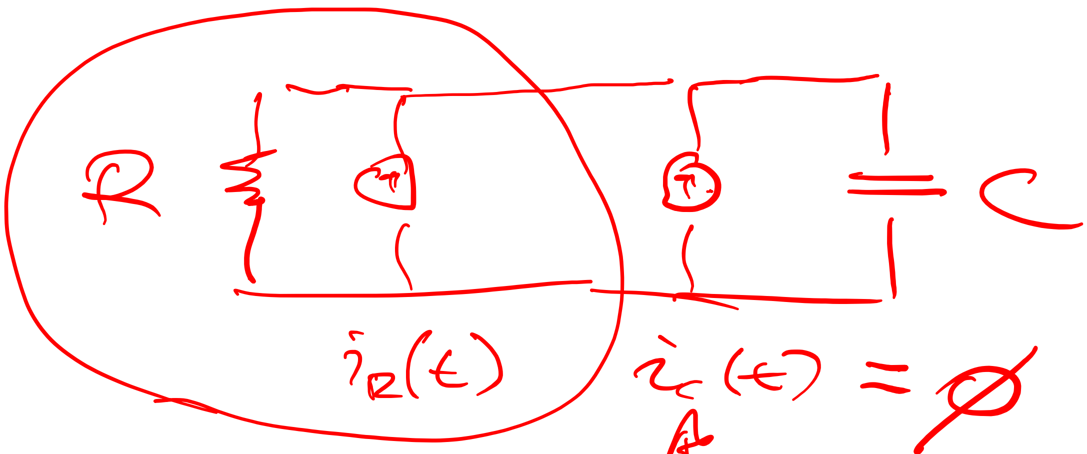




- J. B. Johnson, "Thermal Agitation of Electricity in Conductors," Phys. Rev., pp. 97-109, July 1928.

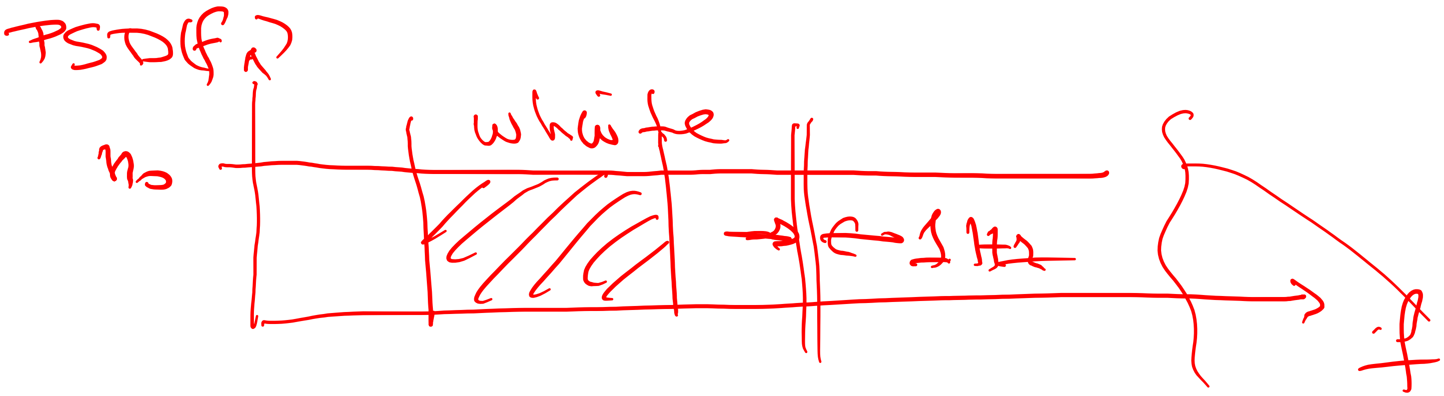
Properties of Thermal Noise

- All "dissipative systems"



* indep of current flow
* zero mean
* iid
* average power

Thermal Noise Power Spectrum



$$PSD(f) = n_0 = 4 k_B \cdot T \qquad \text{THz}$$

$$\approx 2 \cdot 10^{-20} \text{ J} \qquad [\text{W/Hz}]$$

@ 100 °C

noise in Δf

$$P_n = \int_{f_1}^{f_2} PSD \cdot df$$

$$= 4 k T \cdot \Delta f$$

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Modeling Noise

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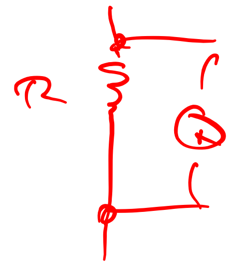
Resistor Noise Models

$$\overline{v_n^2} = P_n \cdot R = 4kTR\Delta f$$



$$\overline{i_n^2} = \frac{P_n}{R} = \frac{4kT}{R} \cdot \Delta f$$

$$R = 1\,k\Omega$$

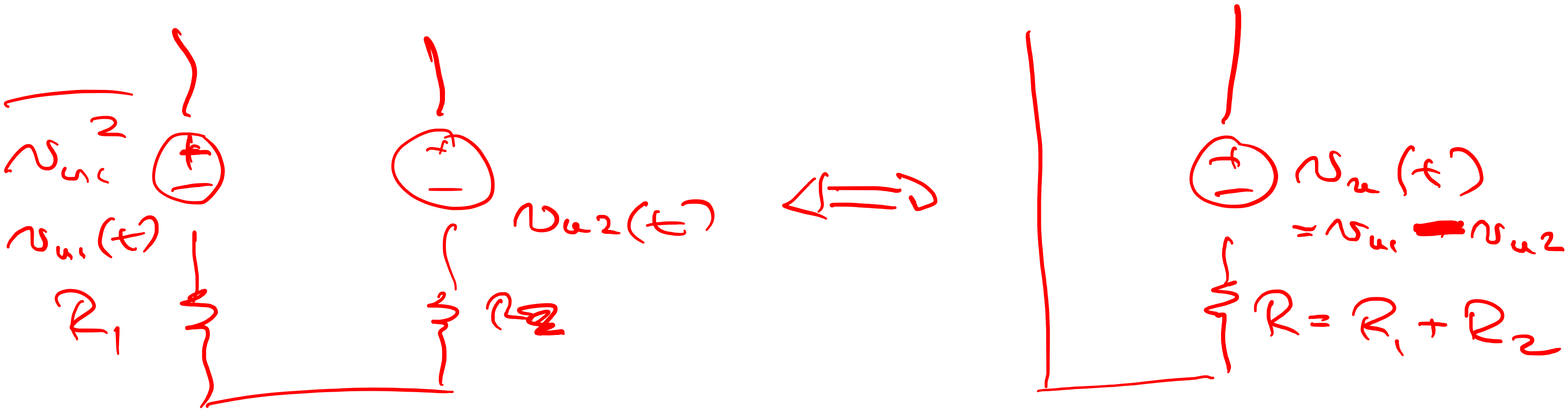
$$\sqrt{\frac{\overline{v_n^2}}{\Delta f}} = 4\frac{nV}{\sqrt{Hz}}$$

$$\Delta f = 1\,MHz \qquad \sqrt{\overline{v_{n,}^2}} = 4\,\mu V \text{ rms}$$

Sloppy Nomenclature

- For convenience, noise in circuits is usually represented by the mean squared noise voltage $\overline{v_n^2}$ or current $\overline{i_n^2}$
- It is customary to refer to these quantities as “noise power”, especially when comparing them to signals which are usually represented by voltages or currents, not power
- The actual noise power is easily obtained by dividing or multiplying the mean squared values by the resistance

Resistors in Series

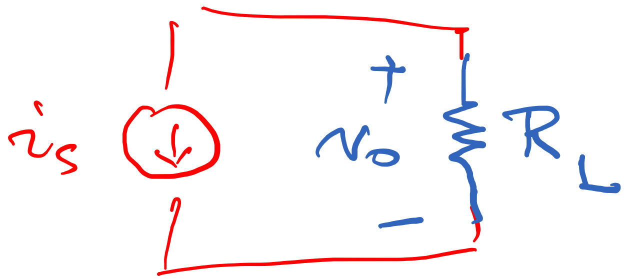


$$\overline{v_n^2} = \overline{(v_{n1} - v_{n2})^2}$$

$$= \overline{v_{n1}^2} + \overline{v_{n2}^2} - \cancelto{0}{\overline{2 v_{n1} v_{n2}}}$$

$$= 4kT \cdot (R_1 + R_2) \cdot \Delta f$$

Signal-to-Noise Ratio



$$P_s = \frac{1}{2} I_s^2 \cdot R_L$$

$$P_n = 4kT \cdot \Delta f \cdot R_L$$

$$SNR = \frac{P_s}{P_n} = \underbrace{\frac{\frac{1}{2} I_s^2}{4kT\Delta f} \cdot \frac{R_L}{R_L}}_{}$$

P_s

60dB 1MHz

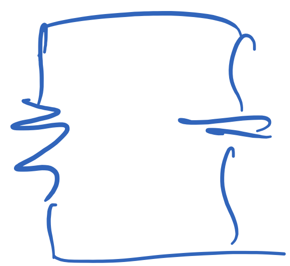
$$P_{s,min} = 4kT \cdot \underbrace{SNR \cdot \Delta f}_{I_n\ f_o}$$

$$= 2\ \text{fnW}$$

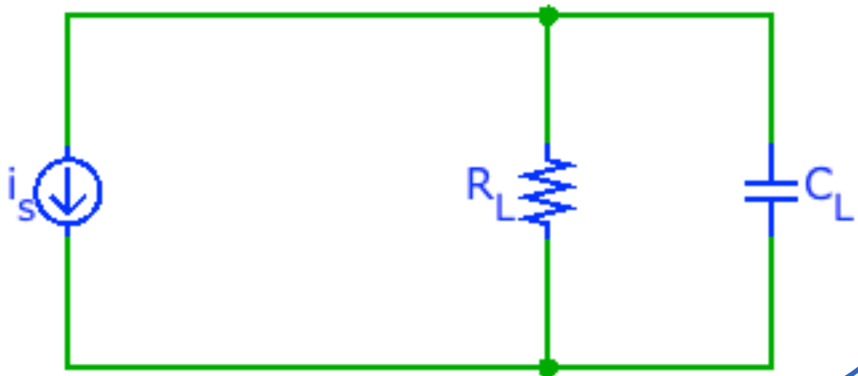
Noise Bandwidth

a) BW set by app ← rec.
human ear

b) Total noise $0 - \infty$ Hz



Band-Limited Noise Example



+

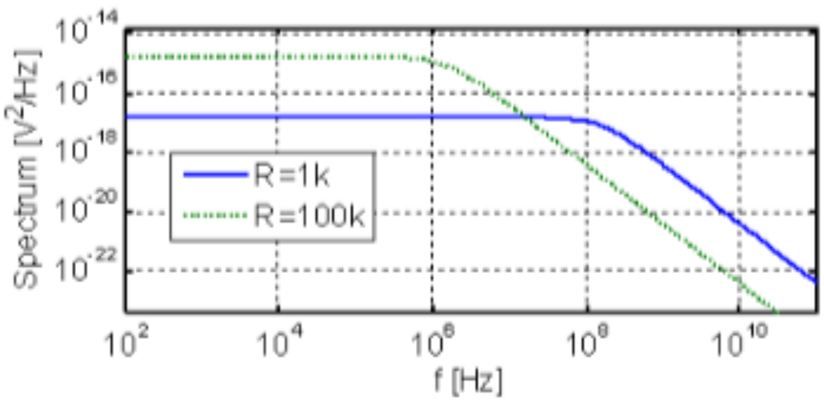
$$\frac{v_o}{i_s} = \frac{R_L}{1 + j\omega R_L C}$$

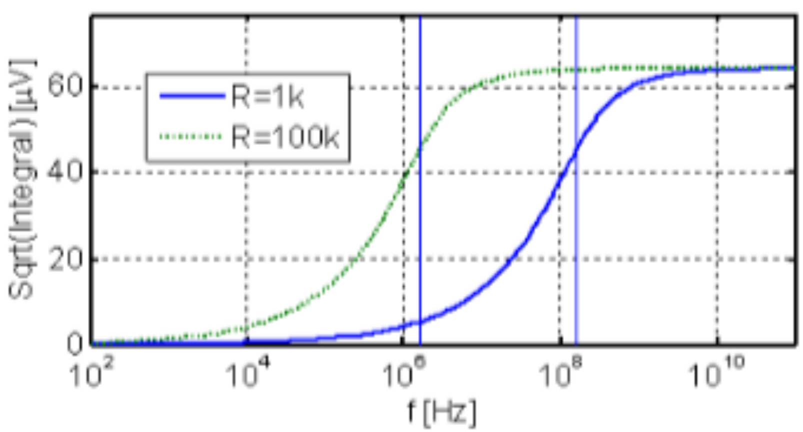
s

PSD $$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f} \cdot \left| \frac{v_o}{i_s} \right|^2$$

$$\overline{v_{o,nT}^2} = \int_0^\infty \frac{\overline{i_n^2}}{\Delta f} \cdot \left| \quad \right|^2 \cdot df = \frac{kT}{C_L}$$

RC Noise Spectral Density





Useful Integrals

$$\int_0^\infty \left|\frac{1}{1+\frac{s}{\omega_o}}\right|^2 df = \frac{\omega_o}{4}$$

$$\int_0^\infty \left|\frac{1}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}}\right|^2 df = \frac{\omega_o Q}{4}$$

$$\int_0^\infty \left|\frac{\frac{s}{\omega_o}}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}}\right|^2 df = \frac{\omega_o Q}{4}$$

Generalization in

A. Dastgheib and B. Murmann, “Calculation of Total Integrated Noise in Analog Circuits,” IEEE TCAS1, vol. 55, no. 10, pp. 2988-2993, Oct. 2008.

Equipartition Theorem

Equivalent Noise Bandwidth

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Noise–Power Tradeoff

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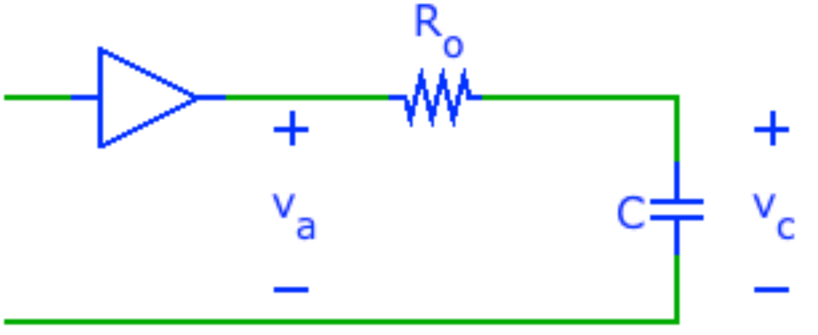
Analog versus Digital SNR

Representative Circuit



Peak SNR versus Capacitance

- SNR versus C for 1-V sinusoidal signal at 100^{o}C

Bits	SNR [dB]	C	
3.0	20	4.1 aF	
6.3	40	412 aF	
9.7	60	41 fF	
13.0	80	4.1 pF	
16.3	100	412 pF	
19.6	120	41 nF	
23.0	140	4.1 μF	

Aside: Oversampling

Noise-Power Tradeoff

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Noise Representations

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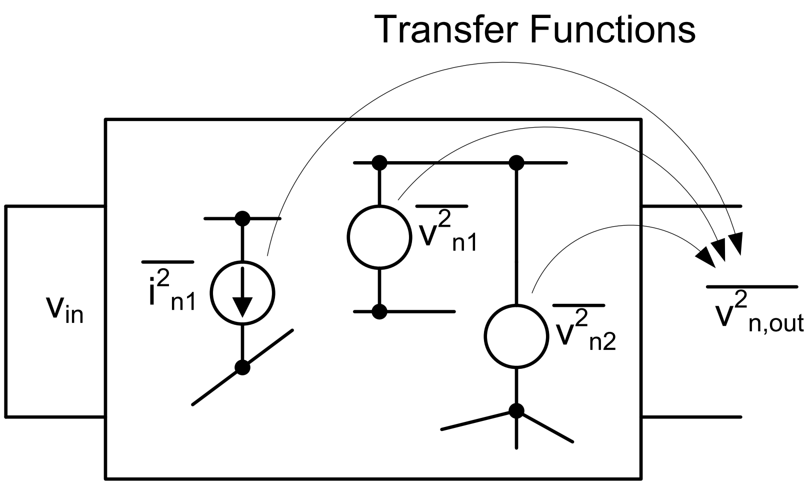
Noise Representations

Output and Input Referred Noise

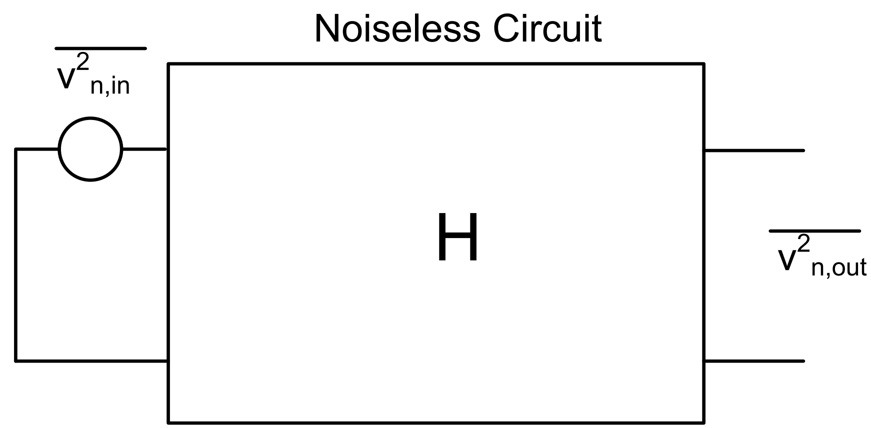
Output



$$v_{out} = H_i \cdot s_{n,i}$$

$$\overline{v_{n,out}^2} = \sum_i |H_i|^2 \overline{s_{n,i}^2}$$

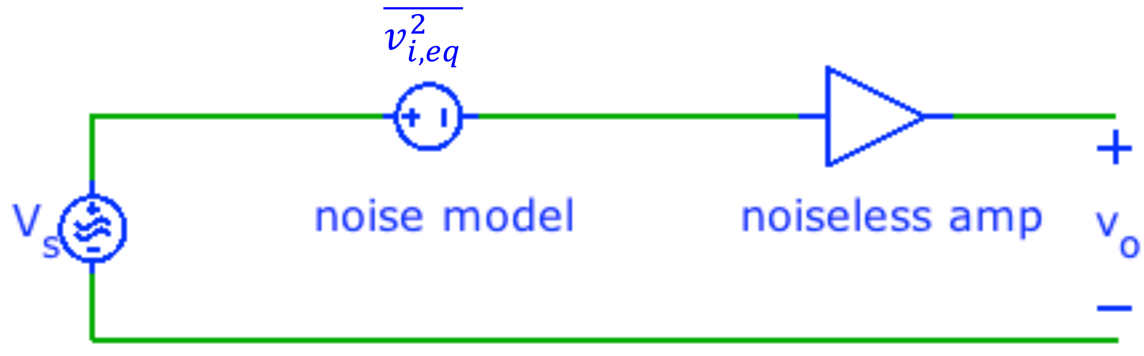
Input



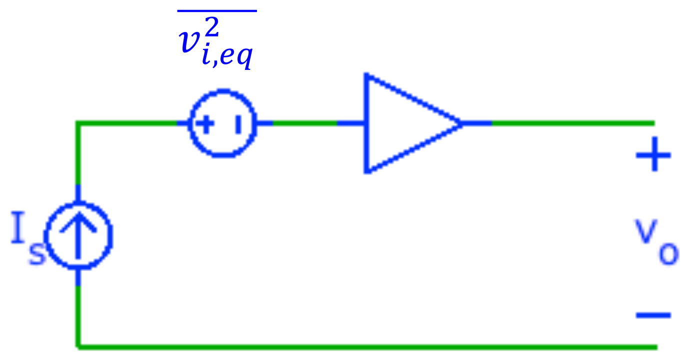
$$v_{out} = H \cdot v_{in}$$

$$\overline{v_{n,in}^2(\omega)} = \frac{\overline{v_{n,out}^2(\omega)}}{|H|^2}$$

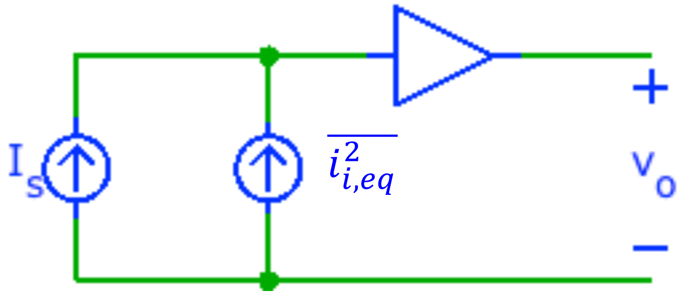
Minimum Detectable Signal



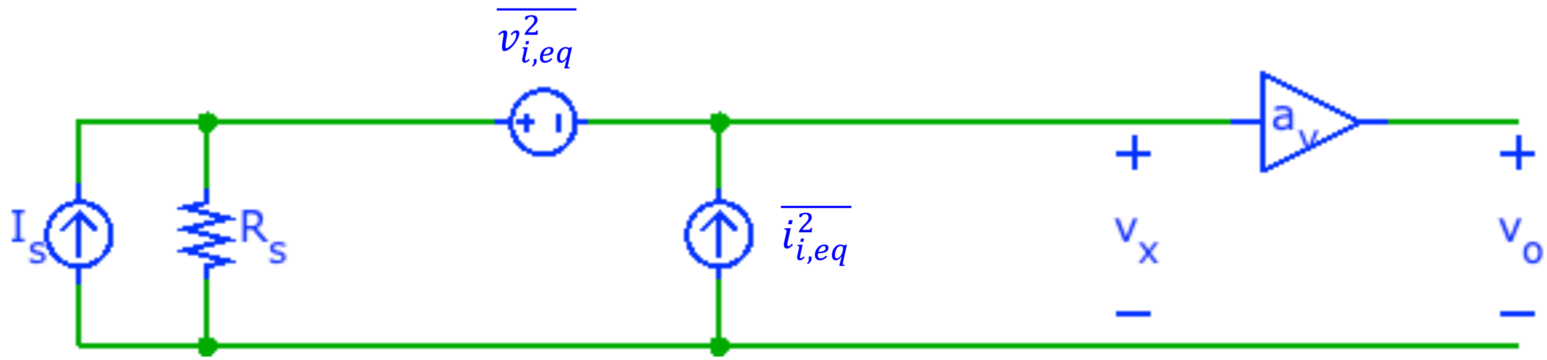
Source Impedance



Equivalent Input Current Noise



Input Voltage and Current Noise Sources



Correlated Noise

- Counting some of noise twice
  - When noise sources are correlated
  - And both noise sources matter (i.e. contribute similarly to the noise at the output of the circuit)

- Situations:
  a) Custom circuit with known R_s:
  Represent input referred noise with one source for a specific R_s
  b) General purpose circuit (e.g. opamp)
  Need to consider correlation in noise calculations *if* both sources matter (rare)

Examples

BJT Opamp

LT1115 - Ultra-Low Noise

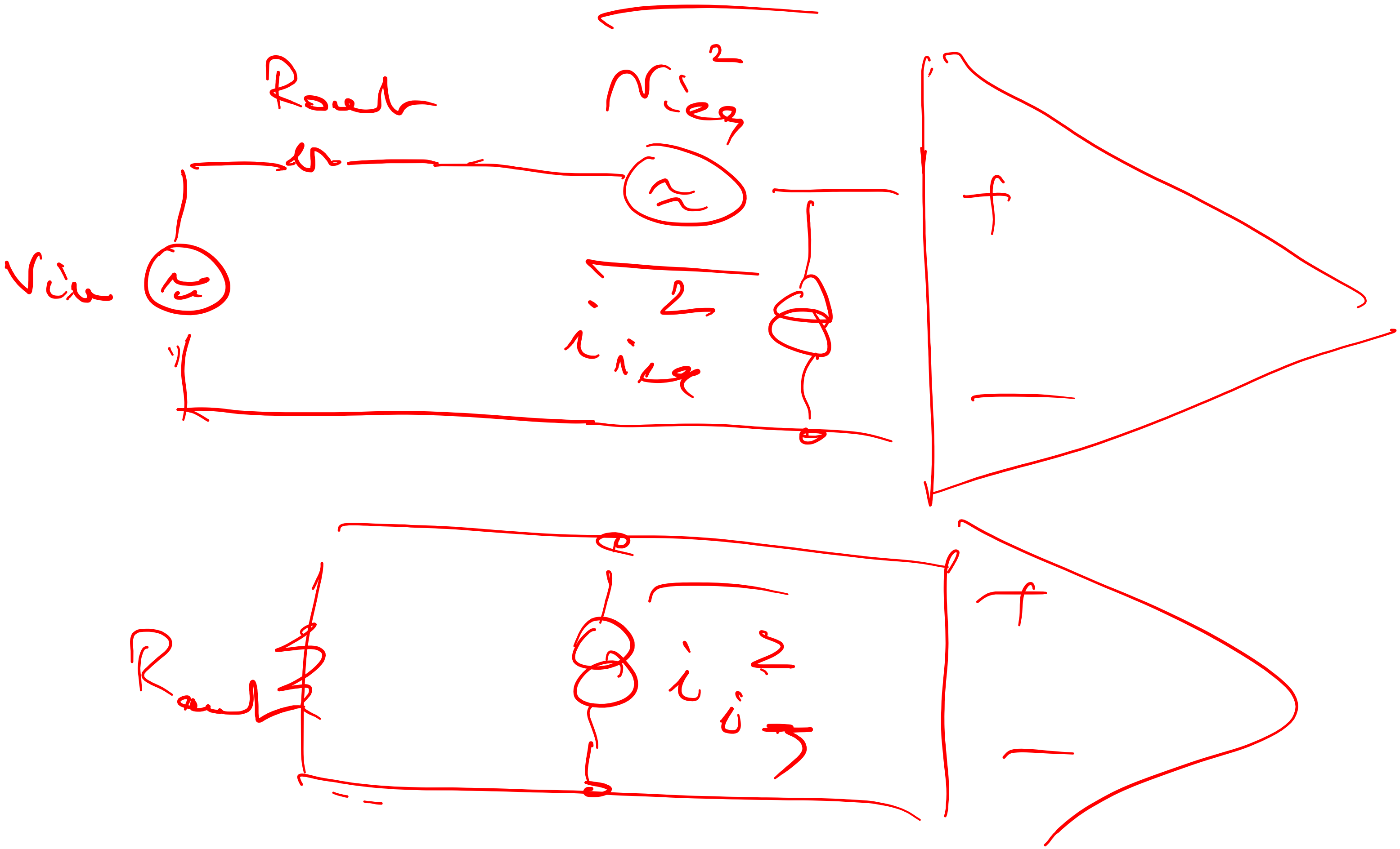
Input Noise Voltage Density	f_0 = 10Hz f_0 = 1000Hz,	1.0 0.9	 1.2	$nV/\sqrt{Hz}$ $nV/\sqrt{Hz}$
Wideband Noise	DC to 20kHz	120		nV_{RMS}
Corresponding Voltage Level re 0.775V		−136		dB
Input Noise Current Density (Note 3)	f_0 = 10Hz f_0 = 1000Hz,	4.7 1.2	 2.2	$pA/\sqrt{Hz}$ $pA/\sqrt{Hz}$

JFET Opamp

OPA827

Input Voltage Noise:		
f = 0.1Hz to 10Hz	250	nV_{PP}
Input Voltage Noise Density:		
f = 1kHz	4	$nV/\sqrt{Hz}$
f = 10kHz	3.8	$nV/\sqrt{Hz}$
Input Current Noise Density:		
f = 1kHz	2.2	$fA/\sqrt{Hz}$

Source Resistance



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Noise Calculations – Example

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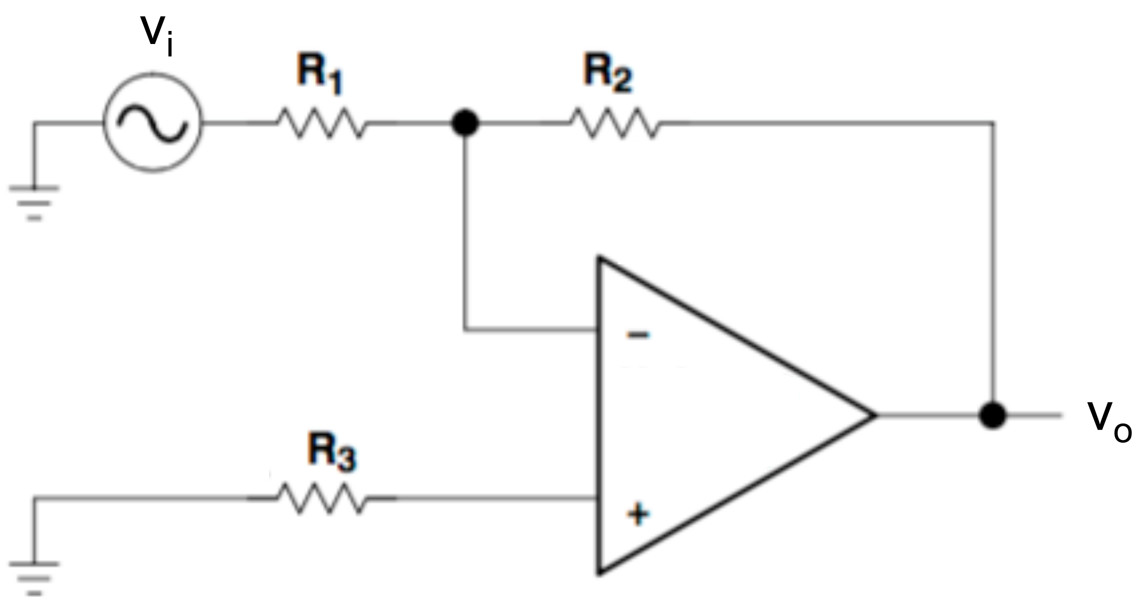
Example: Negative Feedback Amplifier

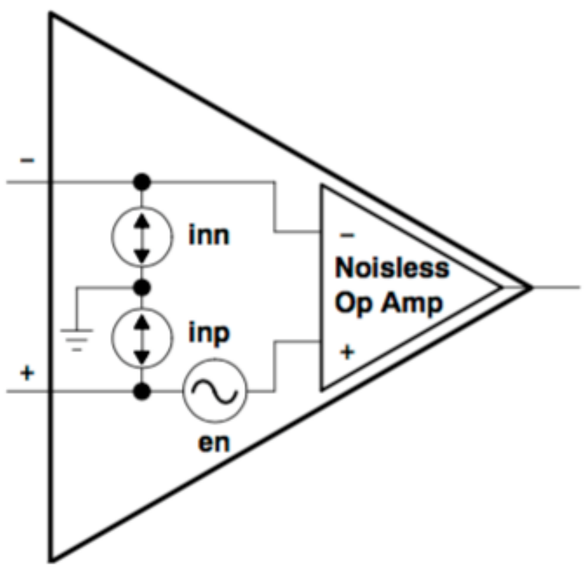


$$v_o = -v_i \frac{R_2}{R_1} = -a_v v_i$$

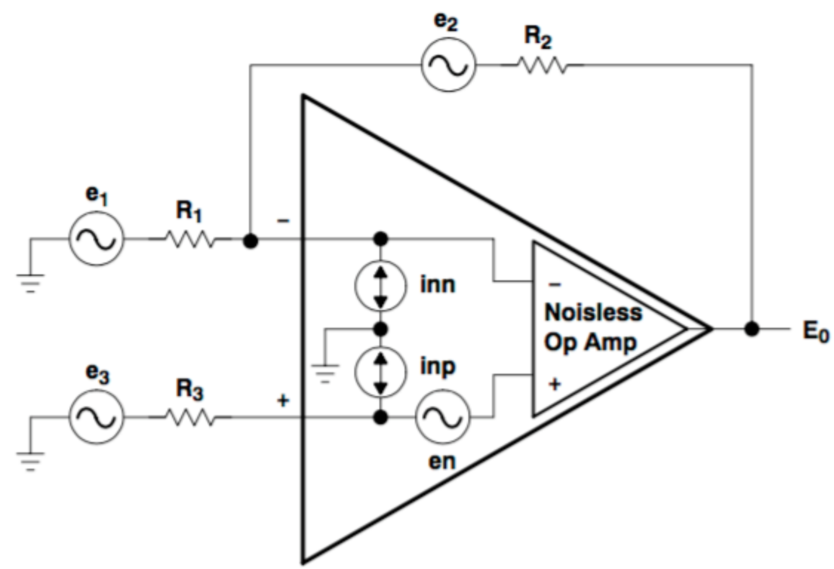
http://www.ti.com/lit/an/slva043b/slva043b.pdf

Noise Model

Opamp



Circuit

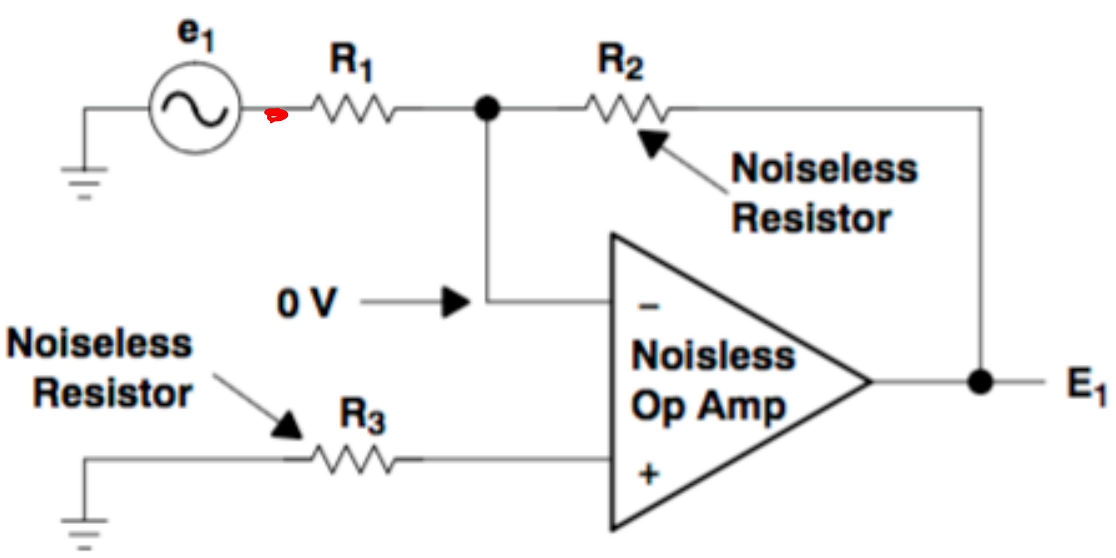


6 noise sources!

Noise Calculation

- One noise source at a time (linear superposition)

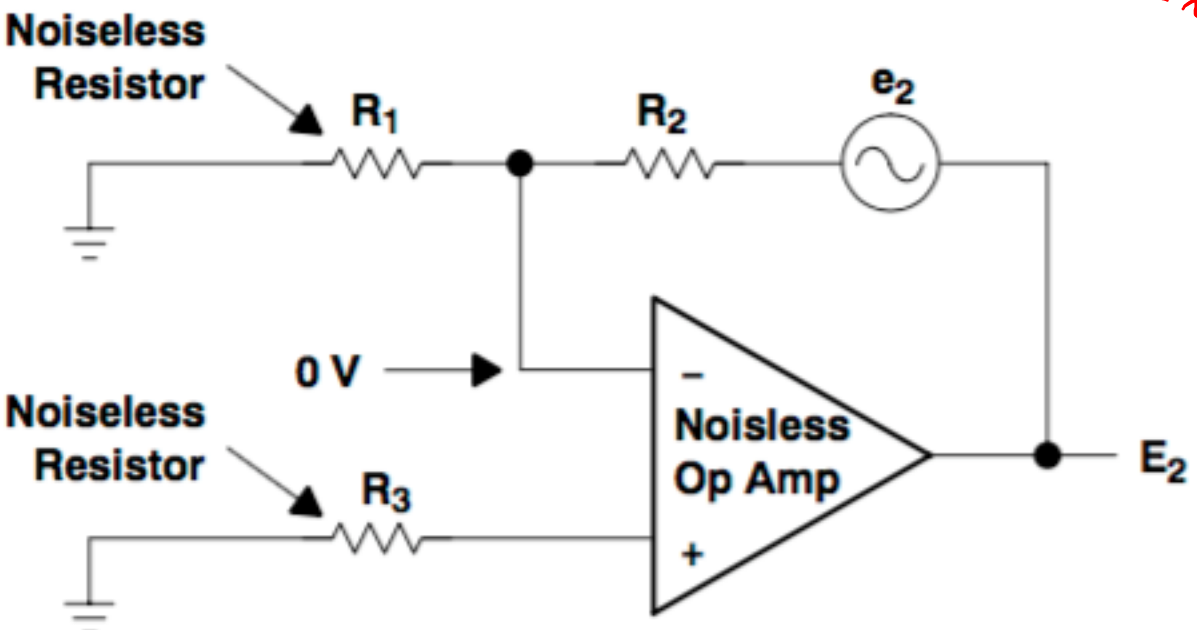
1) Noise at Output from R_1



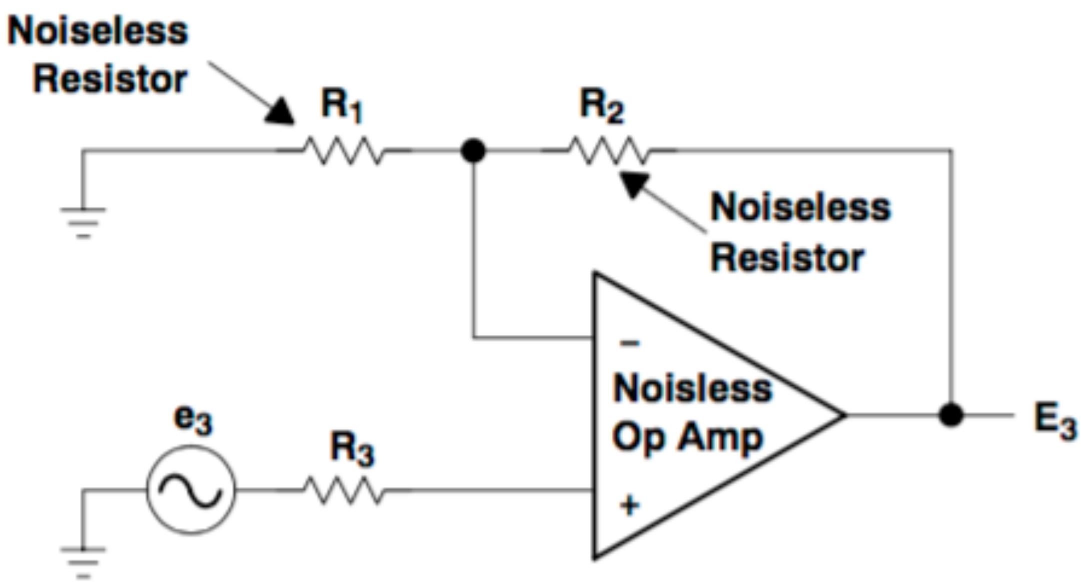
$$\overline{e_{1,o}^2} = \left|\frac{R_2}{R_1}\right|^2 \overline{e_1^2}$$

2) Noise at Output from R_2

$$\overline{e_{n,o}^2} = \overline{e_2^2}$$



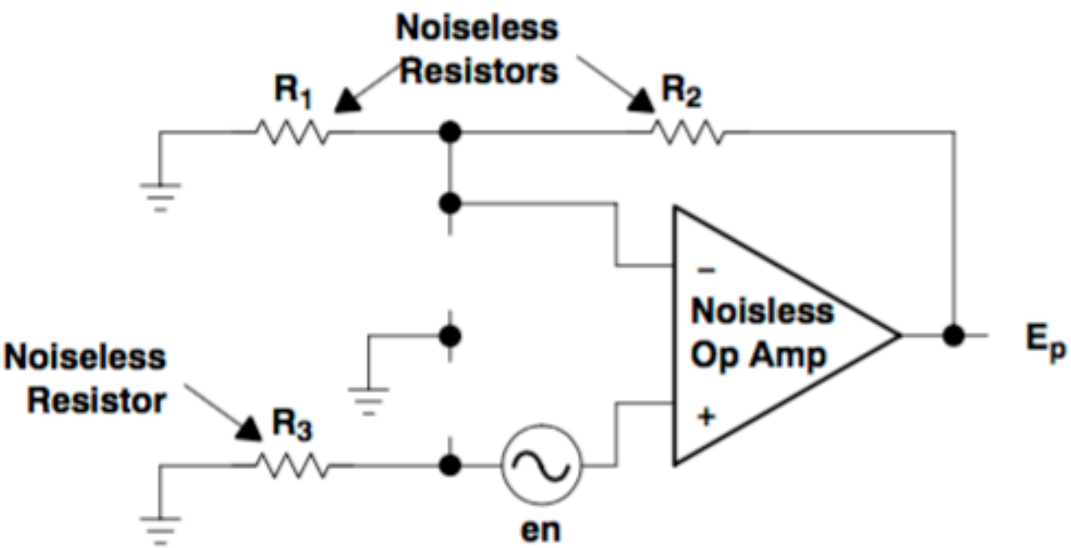
3) Noise at Output from R_3



Output Noise from Feedback Network

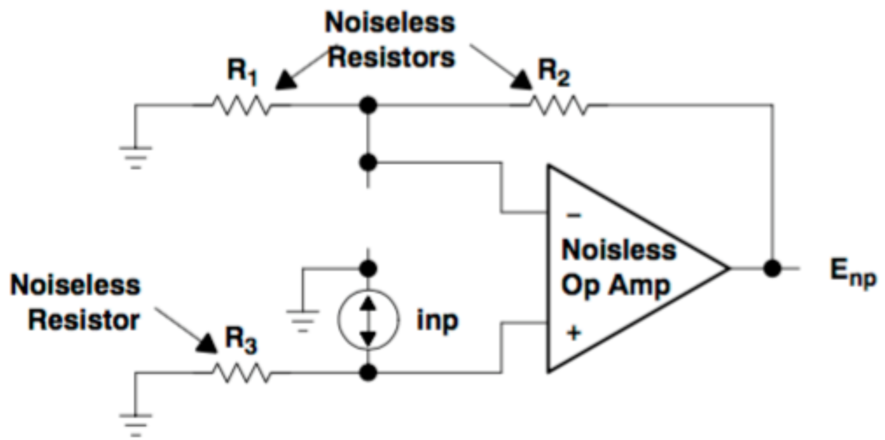
Input Referred Noise from Feedback Network

4) Noise from Opamp Voltage Noise

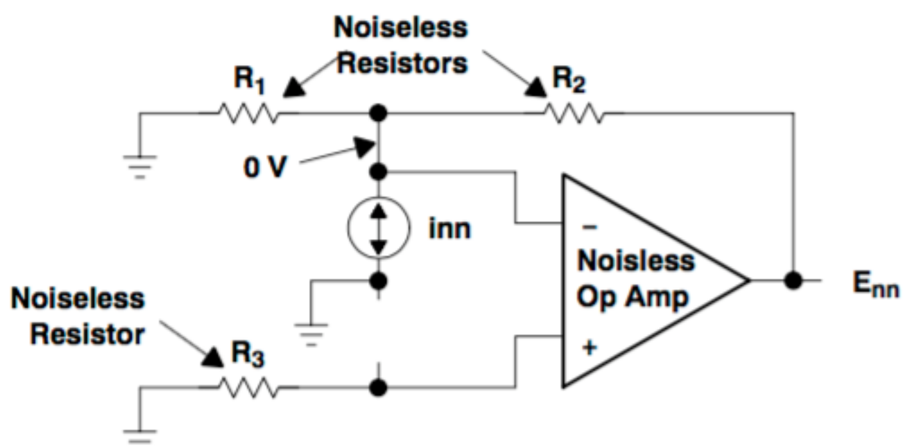


$$\overline{E_n^2} = \int \left[(en) \left(\frac{R_1 + R_2}{R_1} \right) \right]^2 df$$

5) Noise from Opamp Current Noise



$$\overline{E_{np}^2} = \int \left[(i_{np})(R_3)\left(\frac{R_1 + R_2}{R_1}\right) \right]^2 df$$



$$\overline{E_{nn}^2} = \int \left[(i_{nn})(R_2) \right]^2 df$$

Noise from Opamp

$$\overline{v_{oa,n}^2} = \overline{v_{v,n}^2}\left(\frac{R_1 + R_2}{R_1}\right)^2 + \overline{i_{in,n}^2}R_2^2 \qquad \text{for} \quad R_3{=}0$$

$$\overline{v_{if,eq}^2} = \overline{v_{v,n}^2}\left(\frac{1 + a_v}{a_v}\right)^2 + \overline{i_{in,n}^2}\frac{R_2^2}{a_v^2}$$

$$= \overline{v_{v,n}^2}\left(1 + \frac{1}{a_v}\right)^2 + \underbrace{\overline{i_{i,n}^2}R_1^2}_{\text{significant for high } \mathrm{R}_S,\ \mathrm{R}_1}$$

Total Input Referred Noise

$$\overline{v_{i,eq}^2} = \overline{v_{R_1,n}^2}\left(1+\frac{1}{a_v}\right) + \overline{v_{v,n}^2}\left(1+\frac{1}{a_v}\right)^2 + \overline{i_{i,n}^2}R_1^2$$

Source and feedback resistor

Amplifier voltage noise

Amplifier current noise

$\overline{v_{v,n}^2} = 4\frac{\text{nV}}{\sqrt{Hz}}$ $\overline{i_{i,n}^2} = 1.2\frac{\text{pA}}{\sqrt{Hz}}$ (uncorrelated) a_v large

$R_1 = 50\Omega$

$\overline{v_{v,n}^2}$ dominates over $\overline{i_{i,n}^2}$, correlation no concern

$R_1 = 1\text{M}\Omega$

$$\frac{\overline{v_{i,eq}^2}}{\Delta f} = \sqrt{\left(0.9\frac{\text{nV}}{\sqrt{Hz}}\right)^2 + \left(4\frac{\text{nV}}{\sqrt{Hz}}\right)^2 + \left(0.06\frac{\text{nV}}{\sqrt{Hz}}\right)^2}$$

$$\frac{\overline{v_{i,eq}^2}}{\Delta f} = \sqrt{\left(126\frac{\text{nV}}{\sqrt{Hz}}\right)^2 + \left(4\frac{\text{nV}}{\sqrt{Hz}}\right)^2 + \left(1200\frac{\text{nV}}{\sqrt{Hz}}\right)^2}$$

Low source resistance:
Voltage noise dominates
Use BJT

High source resistance:
Current noise dominates
Use MOS

Additional Noise Topics

- Later in EE 240B
  - Noise in sampled data systems
  - (Low) noise amplifier design …
  - Flicker noise
- RF noise metrics (EE 242A)
  - Noise figure
  - Receiver sensitivity
  - Phase noise in oscillators
- Cyclostationary noise
  - Noise in circuits with high signal amplitude which modulates the noise power spectral densities
  - E.g. oscillators, mixers, comparators

Summary

- Thermal noise is fundamental
- Random, but accurately described by universal statistics
- Strong correlation between noise and power dissipation for high accuracy analog systems
  - Up to 4x power for each extra bit
- Noise representations
  - PSD at output
  - Total noise at output
  - PSD at input (depends on R_s)
  - Minimum detectable signal (MDS)
- Noise contributions for different R_s
  - High R_s: current noise dominates (FET advantageous)
  - Low R_s: voltage noise dominates (BJT advantageous)